JET FLOW AROUND A SPHERE

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A sphere placed in a slender gas or liquid jet directed vertically upward is held stably in the jet, but for some relationship of the jet dimensions and the sphere dimension the stability is disrupted and the sphere is ejected by the jet.

It is of interest to clarify the reason for the stable behavior of the sphere in the jet, determine the force maintaining the sphere in the stable equilibrium state, and find the magnitude of the ratio of the jet half-width to the sphere radius for which the sphere is ejected by the jet.

The question of the stability of a sphere in a slender vertical jet in application to the flow about a circle is examined in [1], where the hypothesis was suggested that the jet branching point and convergence point lie on the same diameter. Without this sort of hypothesis the problem does not have a unique solution within the framework of ideal fluid theory.

Jet flow about blunt bodies, particularly the sphere, was examined in [2], where flow past bodies whose dimensions exceeded the dimensions of the nozzle was studied at distances up to 30-40 body diameters from the initial section of the jet. It was established experimentally that there is separation-free flow over a distance up to 8-10 calibers. It is assumed that the separation-free flow in this region is due to branching of the jet into two narrow semibounded jets. As a result of the pressure difference which develops (atmospheric at the outer edge, low pressure at the surface of the body) the jets press close to the surface of the sphere and flow past the surface without separation [3].

In the present study we examined the nature of the flow about a sphere supported in a vertical axisymmetric jet for the case of central flow. The sphere diameters are 74 and 37 mm. The nozzle dimensions vary in the range from 6 to 74 mm.

Generally speaking, the velocity field behind the sphere is defined by four parameters: approaching flow velocity, nozzle radius, body dimension, and distance from the nozzle exit to the section where the front point of the sphere is located. By virtue of the affine nature of the velocity profiles at different sections of the free jet [4], we can take as the characteristic jet width the quantity Y, which is the distance from the axis to the point at which the velocity equals half the axial velocity at the given section.

The experimental results show that the nature of the flow in the wake behind the body in the immediate vicinity of the sphere (0.054 calibers) depends on the ratio Y/R. The value of Y is taken at the section where the frontal point of the sphere is located, and R is the sphere radius.

If $Y/R < 1$ the flow past the body is separation-free, and the velocity profile has the form shown in Fig. 1 (curve 1) for $Y/R = 0.485$. With increasing distance from the body, the profile flattens out and at a distance of 0.5 caliber is similar to the free jet velocity profile. If $Y/R > 1$ the profile has the form shown in Fig. 1a (curve 2) for $Y/R = 1.19$. A reverse flow zone is observed. In this case the dimensions of the recirculation zone increase with increase of Y/R, approaching the dimensions of the zone for uniform flow past the body.

The velocity profile behind the sphere becomes universal for the different nozzles if $Y/R = \text{const}$ (Fig. 1a, curves 1 and 2).

Change of the initial velocity of the approaching stream in the range from 25 to 75 m/sec does not alter the flow pattern for constant Y/R.

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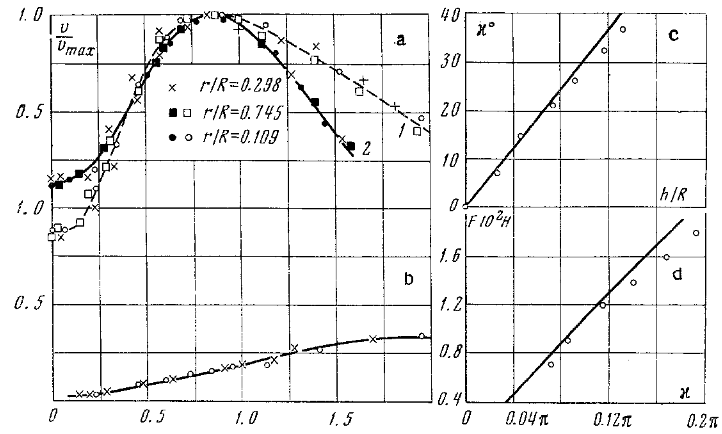

Fig. 1

The kinematics of the motion was investigated and it was found that the flow pattern depends essentially on a single parameter. It is natural to suppose that the dynamics of the motion also depend on this parameter. Experiments with a suspended sphere make it possible to determine easily the sphere drag force, which equals its weight. Using the technique of aerodynamic suspension of the sphere without scales and taking the flow velocity equal to the average velocity at the section where the sphere is suspended, we obtain the sphere drag coefficient

$$\zeta = 8mg / \pi d^2 \rho v^2$$

Here m = sphere mass, d = sphere diameter, v = average velocity, ρ = gas density, and g = gravity acceleration; ζ depends on the dimensionless parameter Y/R.

The drag coefficient is small for $Y/R < 1$. With increase of this parameter, ζ increases monotonically, approaching the value of ζ for uniform flow past a sphere (Fig. 1b). In this case the sphere becomes unstable in the jet and is ejected. It was not possible to establish exactly the value of Y/R for which this phenomenon is observed. The approximate value of the ratio is 2.5–3.0.

Variation of the Reynolds number N_{Re} in the range $6.7 \cdot 10^4 - 2.0 \cdot 10^5$ has very little effect on the drag coefficient, which remains practically constant in this N_{Re} range if Y/R = const.

Now let the axis of the slender jet be shifted relative to the center of the sphere. Then the ideal fluid flow pattern in the vicinity of the jet convergence point must be symmetric to the flow pattern in the jet splitting zone. The jet is deflected by the sphere through some angle, and a stabilizing force directed toward the jet axis is developed which returns the sphere to the stable equilibrium state. The angles of deviation of the jet axis from the vertical behind the sphere for noncentral flow past the sphere (Fig. 1d) were determined experimentally.

If the jet axis passes through the center of the immersed circle, the point of convergence is on the same diameter as the point where the jet branches. It is natural to suppose that, just as in the case of central flow, in the first approximation we can assume that the stagnation points are located on the same diameter if the jet axis is shifted relative to the center of the sphere [1]. The hypothesis of [1] is confirmed by experiments (Fig. 2).

The noted experimental facts, together with the hypothesis of [1], make it possible to construct a model for flow past a sphere which admits approximate calculation. Use of momentum theory makes it possible to find the force which returns the sphere to the stable equilibrium state. It is necessary to know the angle of jet axis deviation from the vertical behind the sphere for noncentral flow about the sphere and the radius of the converging jet.

The hypothesis of [1] makes it possible to find the angle of jet axis deviation from the vertical.

Let us examine the case of slender jet flow around a circle. To find the deviation it is necessary to estimate the distance of the flow branching point from the jet axis. To this end we examine the impact of a jet on a flat plate [5]. For small width of the jet relative to the sphere diameter, in the vicinity of the splitting point we can take the adjacent circular arc to be a straight line tangent to the sphere surface at the point of intersection with the jet axis.

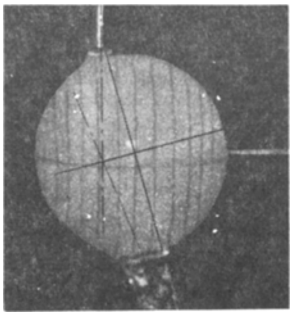

Fig. 2

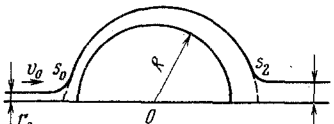

Fig. 3

In this analysis the distance between the flow convergence point and the axis can be written in the form

$$l = 2.3\ \beta d_0, \qquad \beta = {}^1/_2\ \pi - \alpha \tag{1}$$

where d_0 = jet diameter, and α = inclination of jet axis to the surface. This expression is valid for small angles β. Moreover, it follows from the reversibility of the motion that the flow in the vicinity of the point of jet convergence behind the body must be symmetric to the flow in the jet splitting zone; thereby the flow will be symmetric about some straight line which passes through the center of the circle and is perpendicular to the diameter drawn through the stagnation points.

Taking the hypothesis of [1] as the basic assumption and using (1), we can calculate the angle of deviation of the jet axis from the vertical:

$$\varkappa = 2\ [\text{arc sin}\ (h\ /\ R) - \text{arc tg}\ (l\ /\ R)] \tag{2}$$

Here h = shift of jet axis relative to the center of the circle, R = radius of the wetted circle, l = distance from the jet axis to the branching point along an arc of the wetted surface, and $\varkappa$ is the deviation of the jet axis from the vertical.

The geometric pattern is not entirely acceptable for proper determination of the force acting on the sphere, since the viscosity forces cause the leaving jet to be wider than the initial jet. For quantiative estimates it is necessary to account for thickening of the jet. We express the radius of the leaving jet through the initial values: r_0, v_0, ν and R (initial jet radius, approaching flow velocity, kinematic viscosity, and sphere radius). We shall use boundary layer theory. For slender jet flow about a circle the free surfaces of the bifurcated jet can be taken as circles of radius close to the radius of the immersed surface. In the present case the velocity at the outer edge of the boundary layer will be an unknown quantity and therefore we must add to the boundary layer equations the constant discharge equation. The system of equations in the s,n coordinates (s is the longitudinal coordinate along the contour, n is the transverse coordinate, reckoned along the normal to the profile) takes the form

$$v_s \frac{\partial v_s}{\partial s} + v_n \frac{\partial v_s}{\partial n} = \nu \frac{\partial^2 v_s}{\partial n^2} \qquad \frac{\partial v_s}{\partial s} + \frac{\partial v_n}{\partial n} = 0\,, \tag{3}$$

$$G = 2\pi R \sin\frac{s}{R}\int_0^\delta v_s\, dn \tag{4}$$

Using the identity

$$v_n \frac{\partial v_s}{\partial n} = \frac{\partial (v_n v_s)}{\partial n} - v_s \frac{\partial v_n}{\partial n}$$

we transform the first equation (3) to the form

$$\frac{\partial v_s^{\,2}}{\partial s} + \frac{\partial (v_n v_s)}{\partial n} = \nu \frac{\partial^2 v_s}{\partial n^2} \tag{5}$$

We introduce the notation $n/\delta = \eta$, where δ is the thickness of the thin jet flowing over the circle. We integrate (5)

$$\frac{d}{ds}\int_0^\delta v_s^{\,2}\, dn = \nu \left.\frac{\partial v_s}{\partial n}\right|_0^\delta \tag{6}$$

We express the velocity in the form

$$v_s = a\eta + b\eta^2 + c\eta^3\,,\quad \partial^2 v_s\,/\,\partial\eta^2 = 0 \qquad \text{for} \quad \eta = 0, \qquad b = 0$$

We find the coefficient c from the boundary conditions

$$c = -\,{}^1/_3\ a,\ [\partial v_s\,/\,\partial\eta]_{\eta=1} = 0, \qquad v_s = a\,(\eta - {}^1/_3\ \eta^3)$$

Assuming the velocity at the point s_0 equal to the initial velocity $v(s_0) = v_0$, $s_0 \approx r_0$, we write $a(s_0) = {}^3/_2 v_0$ (Fig. 3).

The problem reduces to finding a and δ, for which we have two equations, obtained from (6) and (4) by substitution of the expression for v_s

$$\frac{d}{ds} a^2\delta \int_0^1 \left(\eta - \frac{1}{3}\eta^3\right)^2 d\eta = -\nu \frac{a}{\delta} \tag{7}$$

$$2\pi R \sin \frac{s}{R} \delta a \int_0^1 \left(\eta - \frac{1}{3}\eta^3\right) d\eta = G \tag{8}$$

or

$$\frac{d}{dx}(a^2\delta) = -\frac{\lambda a}{\delta}, \qquad a\delta = \frac{k}{\sin x}, \quad \frac{s}{R} = x, \qquad \frac{315\nu R}{68} = \lambda, \qquad \frac{6G}{5\pi R} = k$$

Excluding δ, we find

$$\frac{d}{dx}\frac{ka}{\sin x} = -\lambda \frac{a^2 \sin x}{k}$$

After some transformations we obtain the differential equation

$$\frac{dy}{dx} = -my^2 \sin^3 x \qquad \left(y = \frac{a}{\sin x}, \ m = \frac{\lambda}{k^2}\right)$$

It has the solution

$$1 / y = m\,(^1/_3 \cos^3 x - \cos x) + C$$

Satisfying the initial condition $a(x_0) = {}^3/_2\, v_0$, $x_0 = s_0/R$, we find a

$$a = \frac{\sin x}{m\,(^1/_3 \cos^3 x - \cos x) + {}^2/_3((s_0 / v_0 R) + m)}$$

Consequently,

$$a_1 = \frac{\sin x_1}{m\,(^1/_3 \cos^3 x_1 - \cos x_1) + {}^2/_3((s_0 / v_0 R) + m)}$$

Assuming that $x_1 = \pi - x_2$ for small x_2, where $x_2 \approx r_1/R \approx s_2/R$, r_1 is the final jet radius, we find a_1 in the form

$$a_1 = \frac{x_2}{{}^2/_3(2m + s_0 / v_0 R)}$$

The discharge for the initial section of the jet is expressed as $G = \pi R_0^2 v_0$, for the final section $G = \pi R^2 x_2^2 \, {}^2/_3 a_1$. On the basis of the constant discharge condition, we equate these two expressions and find the value of the final jet radius in terms of the initial parameters

$$r_1 = \left(r_0^3 + \frac{6.45\nu R^4}{r_0^2 v_0}\right)^{1/3} \tag{9}$$

Applying the momentum conservation theorem and using (2) and (9), we calculate the jet reaction for noncentral flow about a sphere

$$\int \rho \mathbf{v} v_n d\sigma = \mathbf{F}$$

The force acting to return the sphere to the equilibrium state, directed along the x axis, is expressed as

$$F_x = \pi\rho r_0^2 v_0^2 (r_0 / r_1)^2 \sin \varkappa$$

The drag force of the sphere in the jet is

$$F_y = \pi\rho r_0^2 v_0^2 [1 - (r_0 / r_1)^2]$$

The drag coefficient, calculated using the formula

$$\zeta = 8F_y / \rho v_0^2 \pi d^2$$

for the slender jet coincides with the experimental values. The calculated data for the deviation of the jet axis from the vertical and the stabilizing force for different deviation angles agree quite well with the experimental data (Fig. 1c and d).

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